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Analysis of Option Pricing Models

BUS 367
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01

Motivation

Motivation

To test and evaluate whether fundamental options pricing models such as the ***Black-Scholes-Merton (BSM)*** Model as well as the ***Cox-Ross-Rubinstein (CRR)*** Option Pricing Model accurately reflects real market behavior, across different maturities and underlying assets.

02

The Black-Scholes-Merton (BSM) Model

The Black-Scholes-Merton (BSM) Model

def black_scholes(S, K, T, r, sigma, option_type="call"):

```
    d1 = (np.log(S / K) + (r + 0.5 * sigma ** 2) * T) / (sigma * np.sqrt(T))
    d2 = d1 - sigma * np.sqrt(T)


    if option_type.lower() == 'call':
        price = (S * norm.cdf(d1)) - (K * np.exp(-r * T) * norm.cdf(d2))
    elif option_type.lower() == 'put':
        price = (K * np.exp(-r * T) * norm.cdf(-d2)) - (S * norm.cdf(-d1))
    else:
        raise ValueError("option_type must be 'call' or 'put'")

    return float(price)
```

Calculate d1

01 //

Calculate d2

02 //

Price the Option

03 //

03

Cox-Ross-Rubinstein (CRR) Binomial Model

The Cox-Ross-Rubinstein (CRR) Binomial Model

```
def crr_binomial_tree(S, K, T, r, sigma, N, option_type="call"):
    dt = T / N
    u = np.exp(sigma * np.sqrt(dt))
    d = 1 / u
    p = (np.exp(r * dt) - d) / (u - d)
    option_values = np.zeros(N + 1)

    for j in range(N + 1):
        S_T = S * (u**j) * (d**(N - j))
        if option_type.lower() == 'call':
            option_values[j] = max(S_T - K, 0)
        else:
            option_values[j] = max(K - S_T, 0)
    discount = np.exp(-r * dt)

    for i in range(N - 1, -1, -1):
        for j in range(i + 1):
            expected_value = (p * option_values[j + 1] +
                              (1 - p) * option_values[j])
            option_values[j] = discount * expected_value

    return float(option_values[0])
```

Build the Stock Price Tree

01 //

Compute the Final Payoffs at Expiration

02 //

Work Backwards to Price the Option Today

03 //

04

Data Source and Selection

Data Source and Selection Process

Here are the following steps we took to retrieve and clean data in order to test the models:

01

Choose the Underlying Assets

→ AAPL
→ SPY

02

Import Historical Market data from Yahoo Finance

From this, we were able to calculate the S and volatility

03

Import Excel Option Chains Data from NASDAQ and cleanup to a DataFrame

From this, we were able to obtain the other parameters

04

Price Options using the Two Models and Compare to the Midpoint of Bid-Ask

Results were then analyzed and plotted

A Look into the Parameters - November 26th, 2025

The following parameters are what we need to extract from data:

S	K	T	r	σ	N
Spot, Current Underlying Price of Stock	**Strike Price**	**Years to Expiration**	**Risk-Free Rate**	**Annualized Volatility, Sigma**	**Number of Steps, specific to CRR only!**
AAPL = $278.43 SPY = $681.36	A range from ITM to OTM	AAPL: 11/28/2025 SPY: 12/01/2025 12/18/2026	Assumed Constant @ 4%	sigma = daily_ret.std() x np.sqrt(252)	Theoretically, the larger the steps in CRR, closer to BSM!

05

Results & Comparison

Walk Through the Code

Announce your vision for scaling operations.

01

Step title

Define your first goal toward making your business more productive and efficient.

02

Step title

Identify what you'll focus on next to help your team work together and grow your operations.

03

Step title

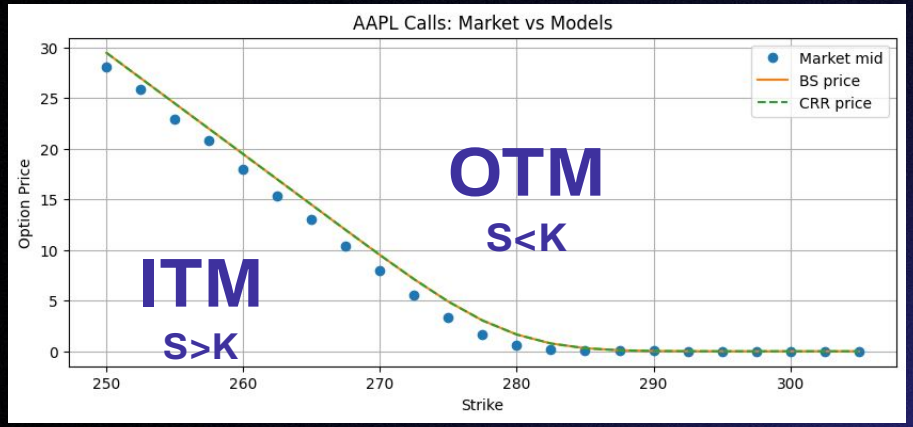
Describe the outcome you want to achieve next. Provide any relevant details about how you'll get there.

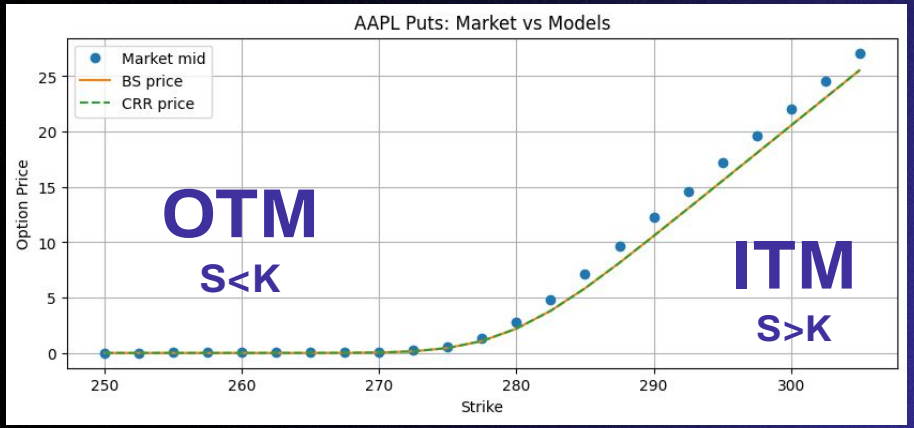
04

Step title

Bring it all together. Share how you'll evaluate, maintain, and advance your operations for long-term sustainability.

AAPL Short-Term Option Chain Results → Option Expiring in 2 Days





$278.43

Current Stock Price of AAPL

$0.29

Mean Absolute Error for Both BSM and CRR

ITM

Call → S>K → exercising gives immediate profit

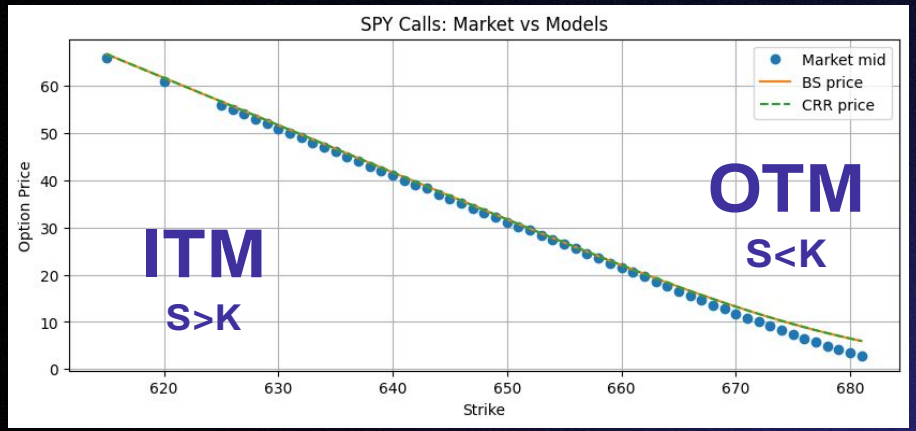
Put → S<K → exercising gives immediate profit

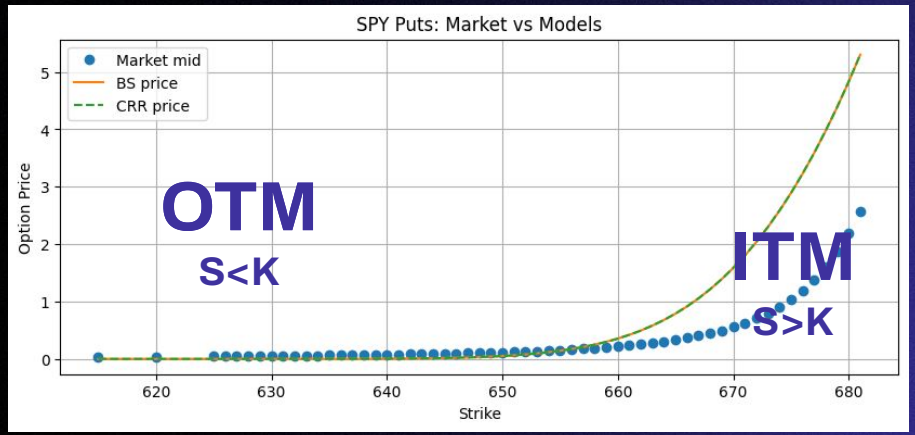
OTM

Call → S<K → wouldn't make sense to exercise

Put → S>K → wouldn't make sense to exercise

SPY Short-Term Option Chain Results → Option Expiring in 5 Days





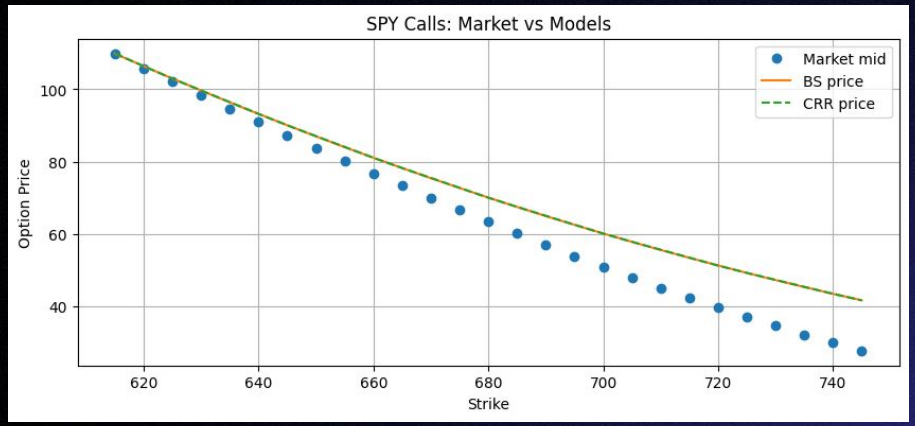
$681.36

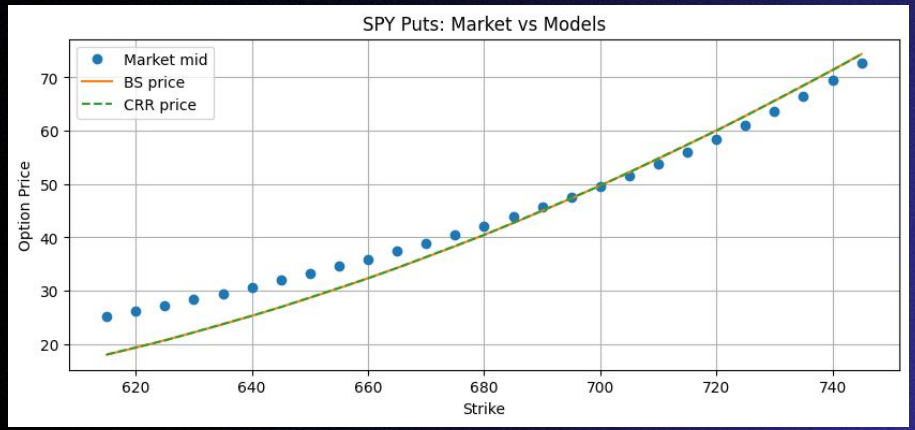
Current Stock Price of SPY

$0.74

Mean Absolute Error for Both BSM and CRR

SPY Long-Term Option Chain Results → Option Expiring in 1 Year





$681.36

Current Stock Price of SPY

$4.94

Mean Absolute Error for Both BSM and CRR

$681.36

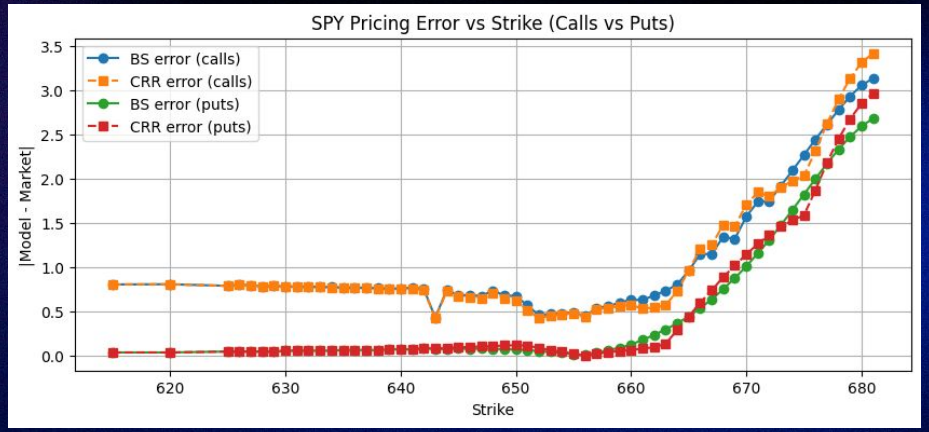
Current Stock Price of SPY

$0.74

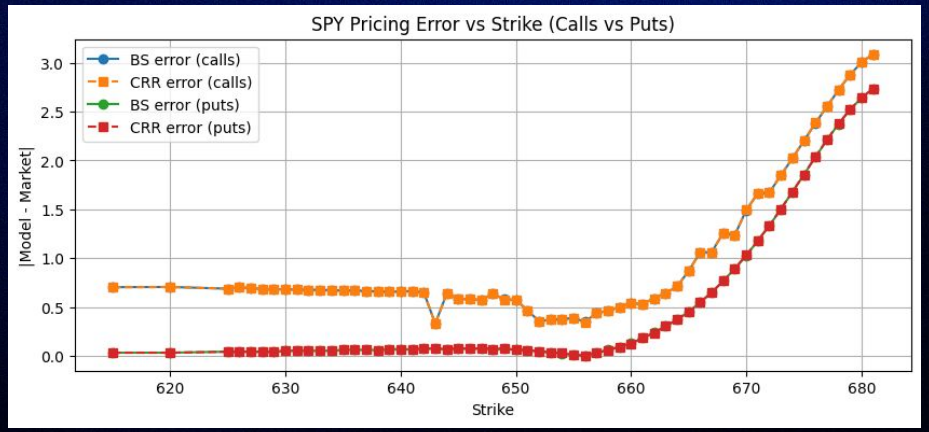
[COMPARED TO SHORTER TERM]

Mean Absolute Error for Both BSM and CRR

Another Interesting Point



N = 5



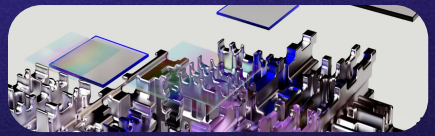
N = 200

06

Discussion & Conclusion

Main Takeaways

The following are the main takeaways from the results:



BSM and CRR perform very well for short-term options

Producing low MAE:

- $0.29
- $0.74

Assumptions (constant volatility, constant interest rate, price behavior) hold reasonably well over a few days/weeks.Short maturities reduce uncertainty.

BSM and CRR DO NOT perform as well for longer-term options

Producing High MAE:

- $4.94

Assumptions may not hold constant in the long-run. Long-term options have large time value and risk premiums, which models cannot capture, causing mispricing.

Increasing CRR Steps (N) makes the model converge with BSM

As the CRR time steps become smaller and smaller ($\Delta t \rightarrow 0$), the binomial model starts to mimic the smooth, continuous price movements assumed by BSM, causing the CRR price to converge to the BSM price.

Thank you!